YIELD CURVE GENERATION

Dr Philip Symes
Agenda

I. INTRODUCTION

II. YIELD CURVES

III. TYPES OF YIELD CURVES

IV. USES OF YIELD CURVES

V. YIELD TO MATURITY

VI. BOND PRICING & VALUATION
Introduction

- A yield curve is a graphical depiction of the relationship between the yield on a class of Securities for different maturities.
Introduction

Types of yield curves (hypothetical):

- **Normal**
  - YTM vs. Maturity

- **Inverted**
  - YTM vs. Maturity

- **Humped**
  - YTM vs. Maturity

- **Flat**
  - YTM vs. Maturity
Yield Curves

Theories Explaining Shapes of Yield Curves:

- Liquidity Preference
  - Investors prefer liquidity — upward sloping yield curve

- Pure Expections Theory
  - Term structure reflects market’s current expectation of future rates

- Market Segmentation Theory
  - Shape is determined by supply of and demand for securities within each maturity sector
  - Shape of the yield curve is best explained by a combination of the three aforementioned theories.
Yield Curves

External Factors Affecting Yield Curves:

- Central Bank Policy
- Inflation Concerns
- Liquidity Desires
- Supply/Demand Conditions.
Types of Yield Curves

Coupon Bearing Yield Curves:

- The coupon bearing yield curve is derived from observable market bond yields at various terms to maturity.

- The “yield to maturity” of coupon bearing Government bonds of various maturities are normally used to construct the coupon bearing yield curve.
Types of Yield Curves

Zero Coupon Rate:

- The zero coupon rate gives the annualised interest rate receivable on a deposit starting immediately where the interest is payable at the maturity date.

- It can be used to calculate the amount that should be placed on deposit now to produce a fixed amount in time, i.e. to calculate the net present value of a single cash-flow.

- It will be higher than the annualised coupon rate on tenors over one year to compensate investors for not receiving any cashflows before maturity.
Types of Yield Curves

Forward Rate Curve:

- The forward interest rate is the interest implied (using arbitrage theory) by the zero coupon rates for periods of time in the future.
- An example of a forward rate is the interest rate for a deposit starting in 3 months time for a period of 3 months.
Types of Yield Curves

Forward Rate Example:

- What is the forward rate for a 3 month deposit starting in 9 months and maturing in 12 months given the following yield curve?

<table>
<thead>
<tr>
<th>Period Months</th>
<th>Zero Coupon Rate %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8.00</td>
</tr>
<tr>
<td>6</td>
<td>8.20</td>
</tr>
<tr>
<td>9</td>
<td>8.30</td>
</tr>
<tr>
<td>12</td>
<td>8.50</td>
</tr>
</tbody>
</table>
Types of Yield Curves

Forward Rate Example (cont.):

Return on a 12 month deposit

\[ = 100 \times (1 + 8.50\%) \]

Return on a 9 month deposit followed by a 3 month deposit

\[ = 100 \times \left( \frac{(1 + (8.30\% \times 274)) \times (1 + (r\% \times 91))}{365 \times 365} \right) \]

Where \( r \) = forward rate for a 3 month deposit starting in 9 months.

\[
 r = \left\{ \left( \frac{(1 + (0.085 \times 365 / 365))}{1 + (0.083 \times 274 / 365)} \right) - 1 \right\} \times (365/91)
\]

\[ = 8.568\% \]

This shows how the marginal forward rate has to rise above the one year rate so that the rate over the first 9 months is raised sufficiently to average 8.5% over a 12 month period.
Uses of Yield Curves

The Importance of Yield Curves:

- Swap valuation requires derivation of the zero coupon yield curve and forward rates.
- Zero coupon rates are used to calculate discount factors while forward rates are used to forecast the floating payments of the swap.
- All three yield curves have an upward sloping shape derived from observable market information.
Uses of Yield Curves

- Separate yield curves exist for:
  - Interbank lending/borrowing rates
  - Yields on government bills/notes/bonds
  - Eurocommercial paper/notes/bonds
  - Swaps (Government Bond yield plus a swap spread or swap rates)
- These reflect the different credit standing and tenor of various borrowers.
Uses of Yield Curves

- Interest rate swaps usually use the swaps yield curve.

- The inputs to swaps yield curves come from different sources.

<table>
<thead>
<tr>
<th>Period (yrs)</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>Interbank deposit rates (LIBOR, BA’s, etc.)</td>
</tr>
<tr>
<td></td>
<td>Interest rate future prices (Dollars, Sterling, French Franc and ECUs)</td>
</tr>
<tr>
<td>1-3</td>
<td>Interest rate futures</td>
</tr>
<tr>
<td></td>
<td>FRA quotes</td>
</tr>
<tr>
<td></td>
<td>Indicative swap rates</td>
</tr>
<tr>
<td></td>
<td>Government bond yields plus a (quoted) swap spread</td>
</tr>
<tr>
<td>3-10</td>
<td>Indicative swap rates</td>
</tr>
<tr>
<td></td>
<td>Government bond yields plus a swap spread</td>
</tr>
<tr>
<td>10-20</td>
<td>Direct telephone quotes of swap rates</td>
</tr>
<tr>
<td></td>
<td>Government bond yields plus a swap spread</td>
</tr>
</tbody>
</table>
Uses of Yield Curves

➢ To derive the zero coupon yield curve the various inputs (BA’s, bond, yields, futures, swap spreads) are utilised.

➢ To calculate the various yield curves (forward rate, zero coupon) we must know:
  - the basis of the various inputs (zero coupon, coupon bearing)
  - compounding frequency (annual, continuous)
  - day count convention (A/360, A/365)

➢ Once we know whether a various input is a zero coupon, bond yield or forward rate one can construct the zero coupon yield curve by using a variety of formulas.
Uses of Yield Curves

- A futures contract that matures in 3 months
  - A 3 month futures contract is a contract on a 3 month deposit starting in 3 months time
    - Forward rate
- A 2 year Government of Canada Bond yield
  - Coupon bearing
- A 5 year swap spread quoted by a broker
  - Coupon bearing
- A 6-12 FRA
  - Forward rate
- A 12 month BA deposit rate
  - Zero coupon
- A 1 year swap rate quote
  - Coupon bearing
Uses of Yield Curves

- In deriving yield curves it is also very important to be able to convert between different coupon bearing rates.

- Rates can be quoted as:
  - continuously compounded
  - Annualised
  - Semi-annual
  - Quarterly
  - simple interest

- Rates can also be quoted on different day count conventions:
  - Actual/360
  - actual/365
  - actual/actual
  - 30/360
Uses of Yield Curves

Example:

- Take a 2-year Government of Canada semi-annual bond yield of 8%:
  
  - Annualised: \( (1 + \frac{r}{2})^2 - 1 \) = 8.16%
  
  - Simple Interest: \( \frac{[(1 + \frac{r}{2})^4 - 1]}{2} \) = 8.493%
  
  - Continuously Compounded:
    \[
    2 \ln (1 + \frac{r}{2}) = 7.844%
    \]
    
    *remember the inverse of Ln(x) is \( e^x \)*

- If the annual yield is 8.16% on an actual/365 basis, what is it on an actual/360 basis?
  
  \[
  8.16\% \times \frac{360}{365} = 8.048\%
  \]
Uses of Yield Curves

Examples:

- 5-year Government of Canada semi annual yield is 10%. What is the quarterly compounded rate?
  
  \[
  \text{Annual Yield} = \left(1 + \frac{10\%}{2}\right)^2 - 1 = 10.25\%
  \]

  quarterly compounded
  \[
  4\left[\left(1.1025\right)^{0.25} - 1\right] = 9.878\%
  \]

  or
  \[
  4\left[\left(1 + \frac{0.1}{2}\right)^{0.5} - 1\right] = 9.878\%
  \]

- A continuous compounded interest rate is 20%, what is the equivalent weekly interest rate?
  
  \[
  52 \left( e^{0.2/52} - 1 \right) = 20.0385\%
  \]

- Annual yield on a bond is 50%, what is the continuously compounded yield?
  
  \[
  \ln \left(1 + 0.5\right) = 40.547\%
  \]
Yield to Maturity:

- In constructing a yield curve one of the inputs used is the yield to maturity of various Government bonds.

- The yield to maturity of a bond is equivalent to its internal rate of return.

- It represents the notional rate of interest at which all cash flows receivable during the life of the bond should be discounted to give the market value of the bond.

- It assumes a flat yield curve.
Credit Spreads

> Counterparties with different credit standing and different tenors of debt will show different yields to maturity as a result.
Yield to Maturity

- A bond with a price of par ($100) and an annual coupon of 8% has a yield to maturity of 8%.
- If the bond price was $110 then the yield to maturity would be less than 8%.
- In simple terms the investment of $110 is yielding $8 in value per year. Therefore its yield as a percentage of the investment is less than 8%.

![Graph showing the relationship between bond price and bond yield/interest rates](image)
Yield to Maturity

Calculation of Yield-to-Maturity:

\[
\text{Bond Price} = C + C + \ldots + 100 + C/(1+Y)^1 + (1+Y)^2 + \ldots + (1+Y)^n
\]

where,

- \( C = \text{Coupon (annual)} \)
- \( Y = \text{Yield to maturity} \)

- To calculate the yield to maturity of a bond requires an iterative process i.e., trial and error.
Yield to Maturity

A three year bond, paying an annual coupon of 10% has a price of $110. What is the yield to maturity?

Try 5%,

\[
\frac{10}{1.05} + \frac{10}{(1.05)^2} + \frac{110}{(1.05)^3} = 113.62
\]

Try 6%,

\[
\frac{10}{1.06} + \frac{10}{(1.06)^2} + \frac{110}{(1.06)^3} = 110.692
\]

Actually it is 6.242%

\[
\frac{10}{1.06242} + \frac{10}{(1.06242)^2} + \frac{110}{(1.06242)^3} = 110
\]
Bond Yields & Values

Bond Yields:

- Why do we need to know yields on bonds?
  - We use them to calculate zero coupon interest rates and therefore the yield curve.

- A 3 year bond with a coupon of 10% and a price of $110 is the same as a 3 year bond with a coupon of 6.242% and a price $100.

- The yield to maturity gives us the equivalent annually compounded coupon of an investment of $100.

- We can use this to construct our yield curve.
Bond Yields & Values

Bond Valuation:

- The mathematics of bond valuation are also very important in deriving a yield curve.

- A bond is just a series of cash flows (coupon and principal). The value of the bond is the discounted value of each cash flow.

- As we have seen by discounting each cash flow at the yield to maturity gives us the bond value.

- We can also value a bond by discounting each cash flow using the "zero coupon" interest rate equivalent to each cash flow’s maturity.
Bond Yields & Values

Example:

Bond Maturity = 3 years
Coupon = 10% annually

Zero Coupon Rates

<table>
<thead>
<tr>
<th>End of Year</th>
<th>1</th>
<th>7%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>9%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11%</td>
</tr>
</tbody>
</table>

Bond Value = \(10 + 10 + 110 = 98.19\)

\[
\text{Bond Value} = 10 \times (1.07) + 10 \times (1.09)^2 + 110 \times (1.11)^3
\]
Derivation of a Yield Curve:

- Using bond yields and BA deposit rates we are going to derive a zero-coupon yield curve.
- The zero coupon yield curve can then be used to calculate forward rates and discount factors which will then be used to value a swap.
- As we have already seen the inputs are used by valuation models such as Oberon to calculate a zero coupon yield curve.

**Diagram:**

- Future prices
- LIBOR rates (annual + actual/360)
- Bond yields (semi-annual + A/365)
- Swap rates

**Output:**

- Zero-coupon yield curve (annual + actual /365 basis)
Example:

The inputs are:

**BA Deposit Rates (A/360)**

<table>
<thead>
<tr>
<th>Period</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 month</td>
<td>5.346%</td>
</tr>
<tr>
<td>3 months</td>
<td>5.395%</td>
</tr>
<tr>
<td>6 months</td>
<td>5.494%</td>
</tr>
<tr>
<td>9 months</td>
<td>5.573%</td>
</tr>
<tr>
<td>12 months</td>
<td>5.622%</td>
</tr>
</tbody>
</table>

**Bond Yields plus swap spreads (semi-annual, A/365)**

<table>
<thead>
<tr>
<th>Period</th>
<th>Bond Yield</th>
<th>Swap Spread</th>
<th>Add-in</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 year</td>
<td>5.86%</td>
<td>0.04%</td>
<td>5.9%</td>
</tr>
<tr>
<td>3 year</td>
<td>6.13%</td>
<td>0.04%</td>
<td>6.17%</td>
</tr>
<tr>
<td>4 year</td>
<td>6.34%</td>
<td>0.06%</td>
<td>6.40%</td>
</tr>
<tr>
<td>5 year</td>
<td>6.54%</td>
<td>0.10%</td>
<td>6.64%</td>
</tr>
</tbody>
</table>
Bond Yields & Values

- Conversion of Semi-Annual Yields to Annual and A/360 to A/365
- The bond yields and swap spreads are quoted on a semi-annual basis. The initial step (for simplicity) is to convert these to an annual yield as follows:

**2 year rate**

\[
\left(1 + \frac{0.059}{2}\right)^2 - 1 = 5.987\%
\]

- The BA deposit rates are quoted as A/360, therefore they need converting to A/365

**1 month rate**

\[5.346\% \times \frac{365}{360} = 5.42\%\]

**Converted Rates:**

**Bond Yields and Swap Spreads**

- 2 year: 5.987%
- 3 year: 6.265%
- 4 year: 6.502%
- 5 year: 6.75%

**BA Deposit Rates**

- 1 month: 5.42%
- 3 months: 5.47%
- 6 months: 5.57%
- 9 months: 5.65%
- 12 months: 5.70%
Bond Yields & Values

Boot-Strap Technique

- The BA deposit rates are now in zero coupon format and on an A/365 basis. No further calculation is required.
- Beyond 12 months we have coupon bearing yields on an annual A/365 basis.
- The method we use to calculate zero coupon rates beyond one year is called the boot-strap technique.
  - It is similar to the method we used to value a bond.
- As you remember the formula for valuing a bond was:

\[
\text{Price} = \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \ldots + \frac{C}{(1+r_n)^n} + P
\]

where,

- \(C\) = Coupon
- \(r\) = Zero coupon rate at each maturity
- \(P\) = Principal
We also know that the bond yield is the coupon which returns a price for the bond of $100 (par).

We can now use this formula to calculate zero coupon rates beyond one year.

2 year point

\[
\begin{align*}
\text{Coupon} &= 5.987\% \\
\text{Price} &= \$100
\end{align*}
\]

\[
5.987 + 105.987 = 100
\]

\[
(1+r_1)(1+r_2)^2
\]

Rearranging formula,

\[
100 - 5.664 = 105.987
\]

\[
(1+r_2)^2 = 105.987
\]

\[
(1+r_2)^2 = 94.336
\]

\[
r_2 = (1.1235^{0.5}) - 1
\]

\[
r_2 = 5.995\%
\]
The same technique applies to 3, 4 and 5 years.

Our zero coupon yield curve (A/365 is as follows:

- 1 month  –  5.42%
- 3 months –  5.47%
- 6 months –  5.59%
- 9 months –  5.65%
- 12 months – 5.70%
- 2 year    –  6.00%
- 3 years   –  6.29%
- 4 years   –  6.55%
- 5 years   –  6.82%
Bond Yields & Values

More Advanced Techniques:

- The zero-coupon rates we have calculated are for specific dates. If we required a rate between two points the easiest method is to linearly interpolate.
- Yield curve generators such as Oberon have more advanced methods of interpolation between rates and calculating yield curves.
- Log-linear interpolation and “Cubic Spline”: These introduce a curve between two points rather than a straight line.
Bond Yields & Values

Which Instruments to Use Depends on the Curve Being Constructed

- Most liquid instruments
  - indicates real market
  - probably most usable as hedges
- Relevant credit rating/Index type
  - LIBOR based instruments
  - Government based instruments
- Quoted IR Products
  - Cash (Libor or BA loans and deposits)
  - Futures (e.g. LIFFE traded contracts) and FRA’s
  - Commercial Paper and CD’s
  - Government Stock (e.g. T-Bills and Gilts)
  - Swaps (if liquid secondary markets)
Yield curves are widely used to price bonds and other interest rate products. Yield curves are graphical descriptions of the relationship between interest rate payments and maturity. There are different yield curves for coupon bonds, zero bonds and forward rate products. Yield-to-maturity is defined as the percentage return on a product if held to maturity. This assumes no reinvestment risk of coupon payments. Yield curves are used for comparing the price payoff of different products. I.e. for pricing products at different maturities. Different methods exist for interpolating yield curves to price products.